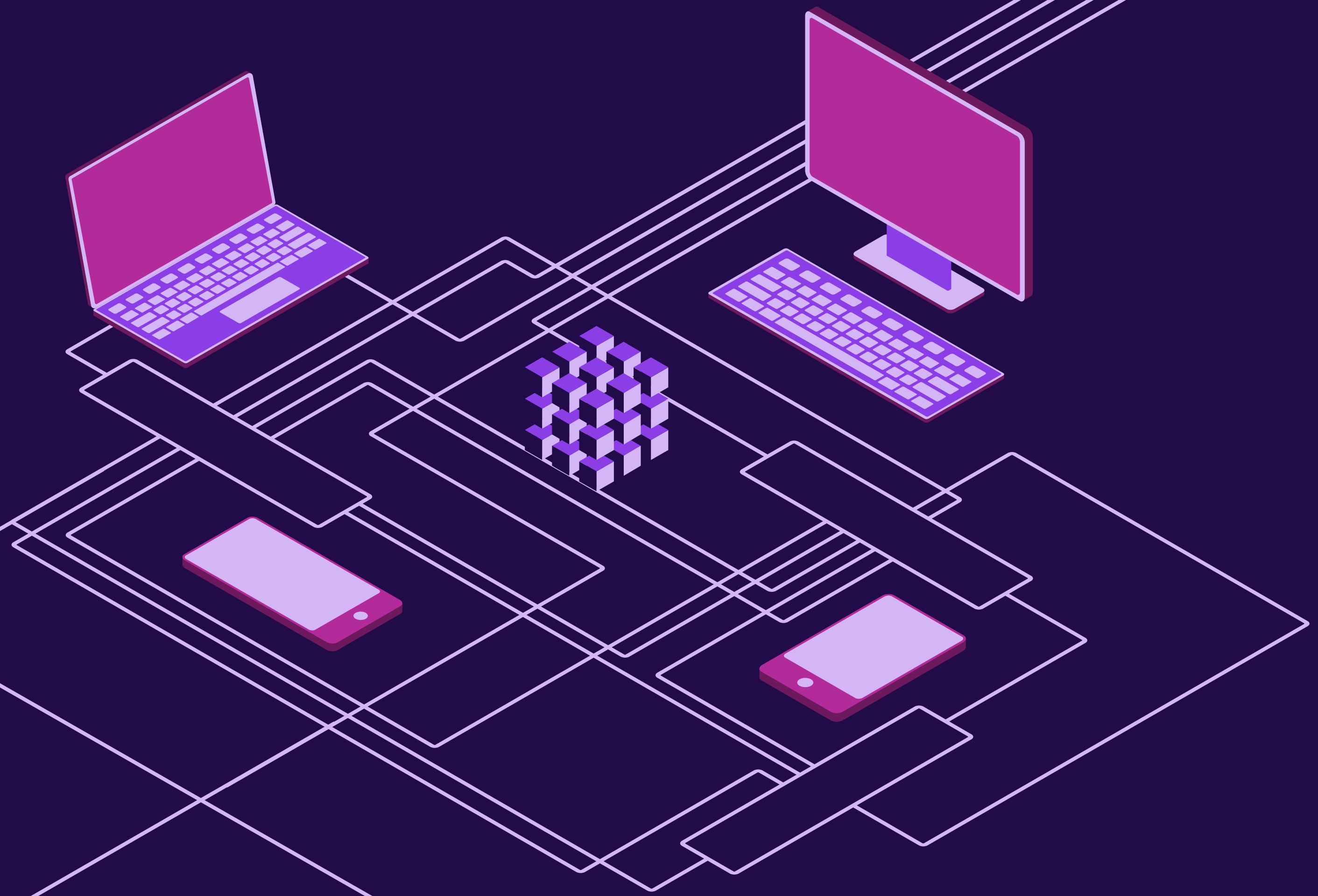
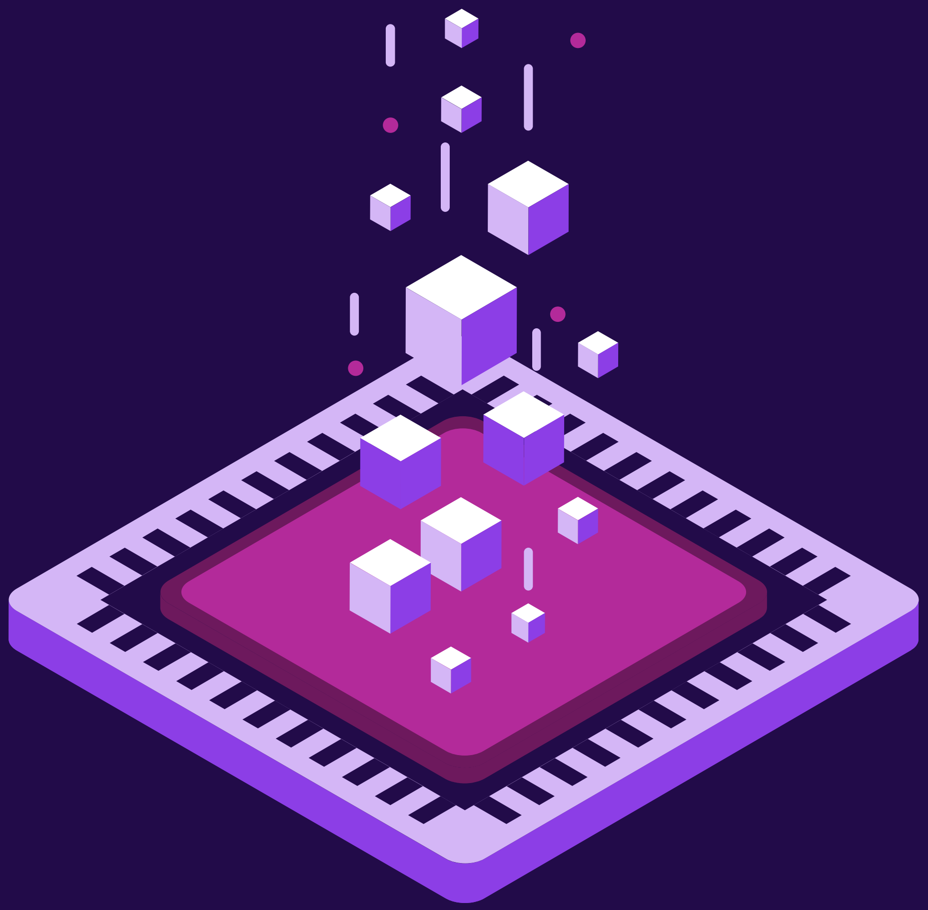
LOGISTIC REGRESSION

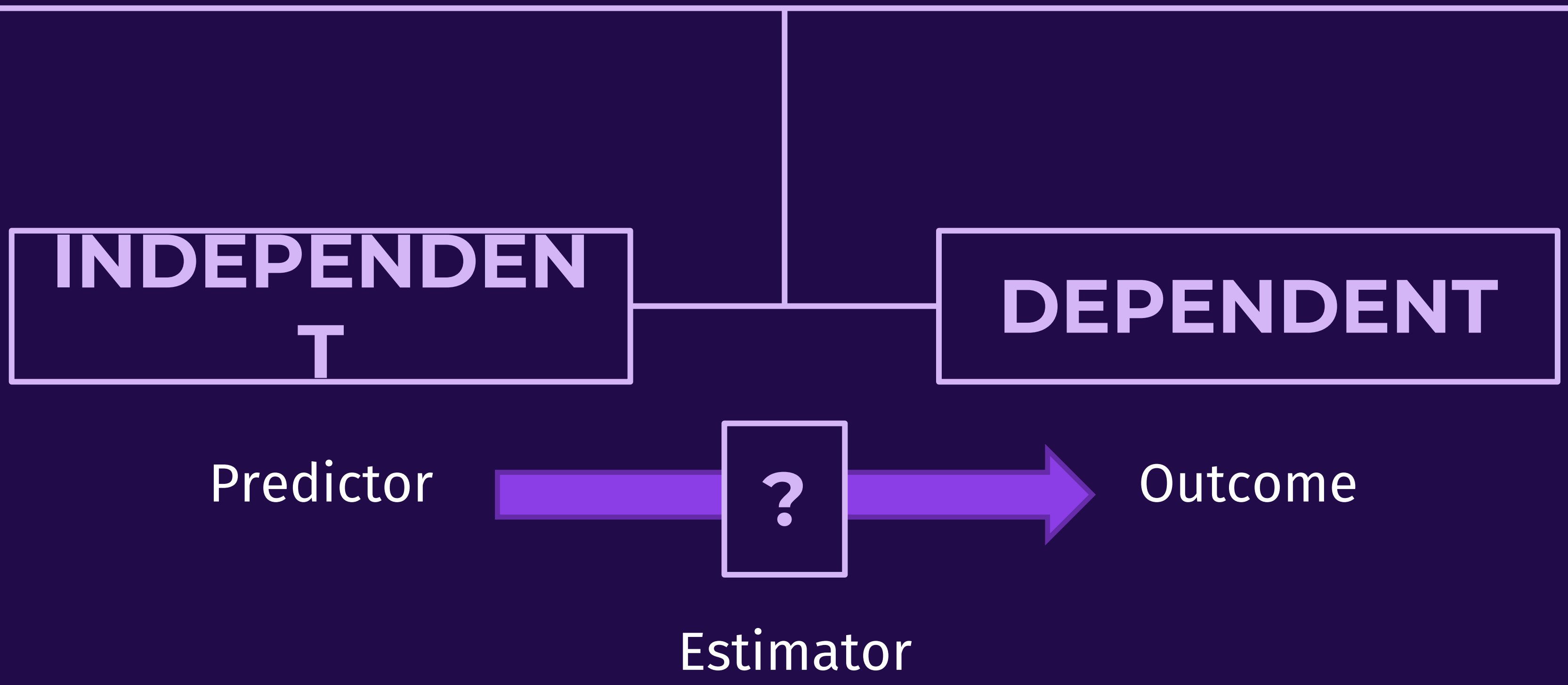
IN CLASSIFICATION MODEL





WHAT IS REGRESSION?

Regress -> estimate **relationship** between variables


INDEPENDEN
T
DEPENDENT
Predictor
?
Outcome
Estimator

SOME TYPES

Linear

Assume linear relation

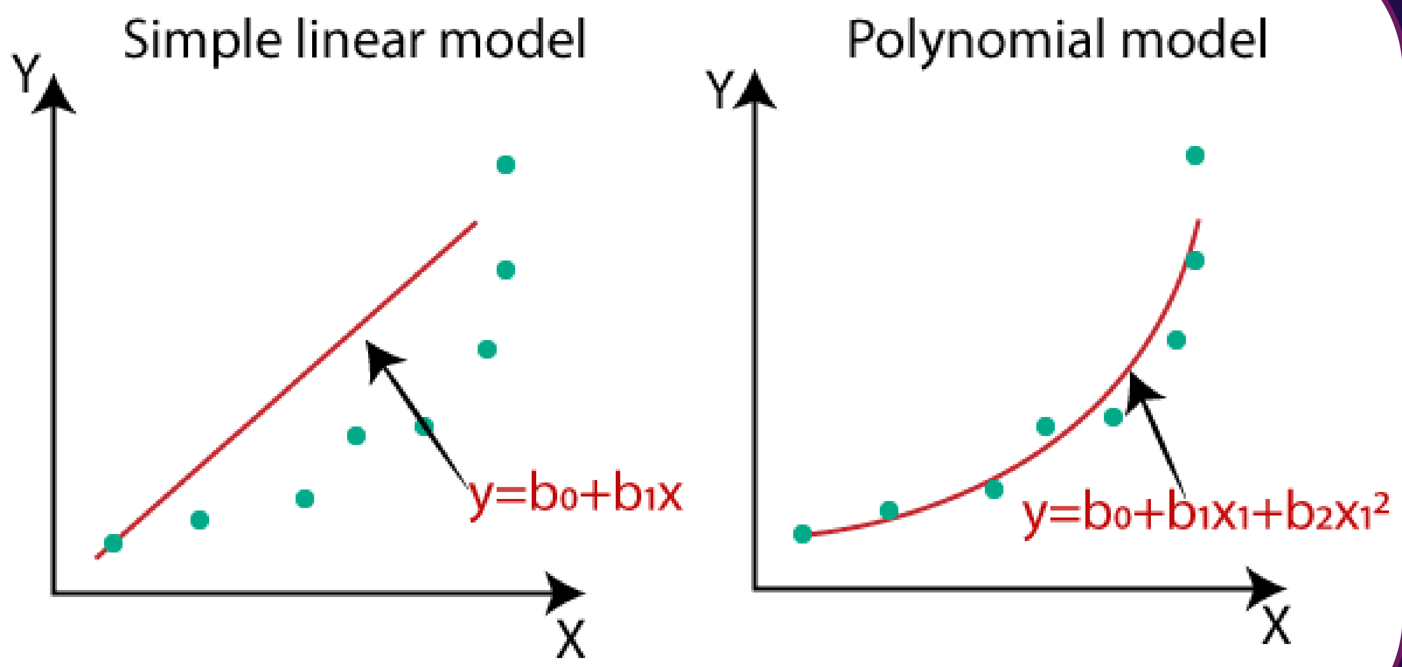
Polynomial

Assume polynomial relation

Logistic

Assume logistic relation

EXAMPLE



Source: Javatpoint.com

WHAT IS LOGISTIC?

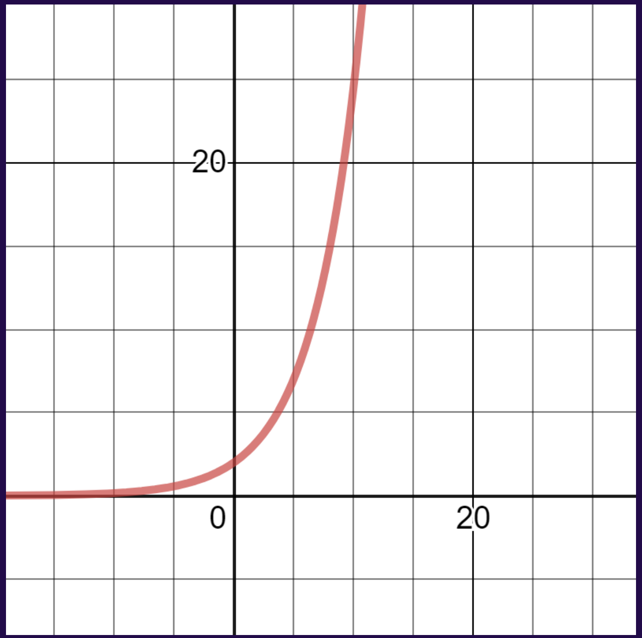
- Pierre Verhulst (1845-1847) developed a model of bounded population growth
- Verhulst named it “logistic model”, the reason is unknown.
- One guess is that this model can be used to predict the supplies an army requires

$$\frac{dy(x)}{dx} = a \cdot y(x)$$

$$\frac{dy(x)}{dx} = a \cdot y(x)(b - y(x))$$

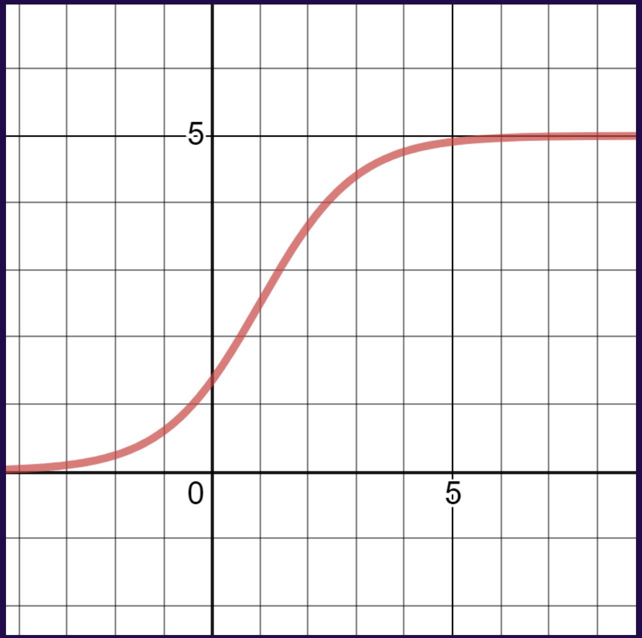
Simple growth

Logistic



Simple growth

$$y(x) = x_0 e^{kx}$$



Logistic

$$y(x) = \frac{b}{1 + e^{a(x - x_0)}}$$

WHY MUST LOGISTIC?

- Probability function must be $(-\infty, \infty) \rightarrow [0,1]$
- Using polynomial regression would be computationally expensive
- One of the simplest functions met the criteria is logistic function

BINARY CLASSIFICATION

We need a model that estimate some predictors to probability of of Bernoulli outcome

INDEPENDEN
T

DEPENDENT

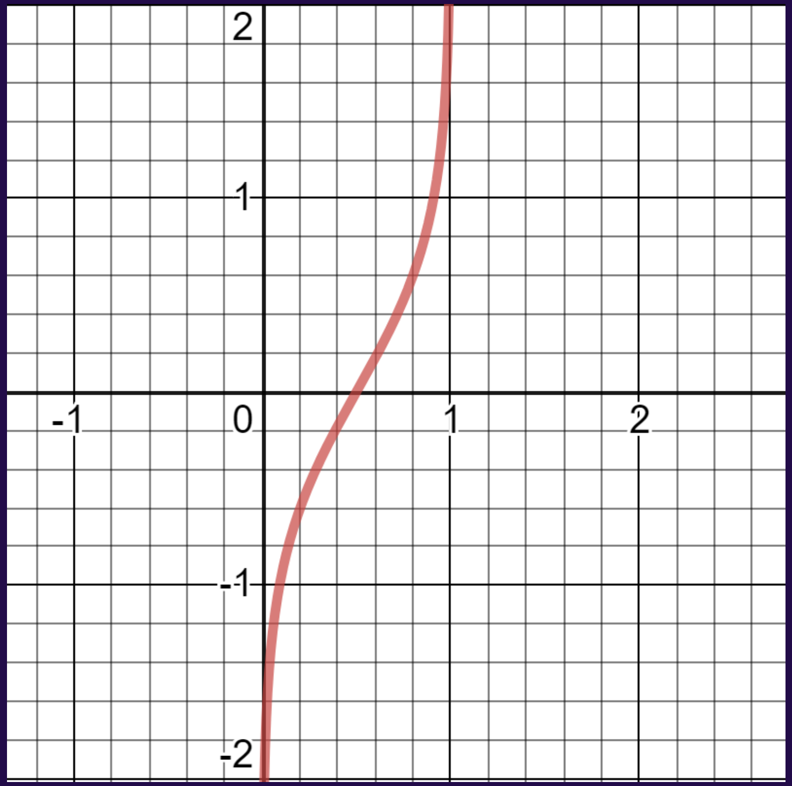
Some features
$x_1, x_2, \ldots$

?

Estimator

Probability of a Bernoulli Outcome
$P(Y = 1)$

$$\mathrm{P(Y = 1)} = \Theta(x_1, x_2, \ldots)$$

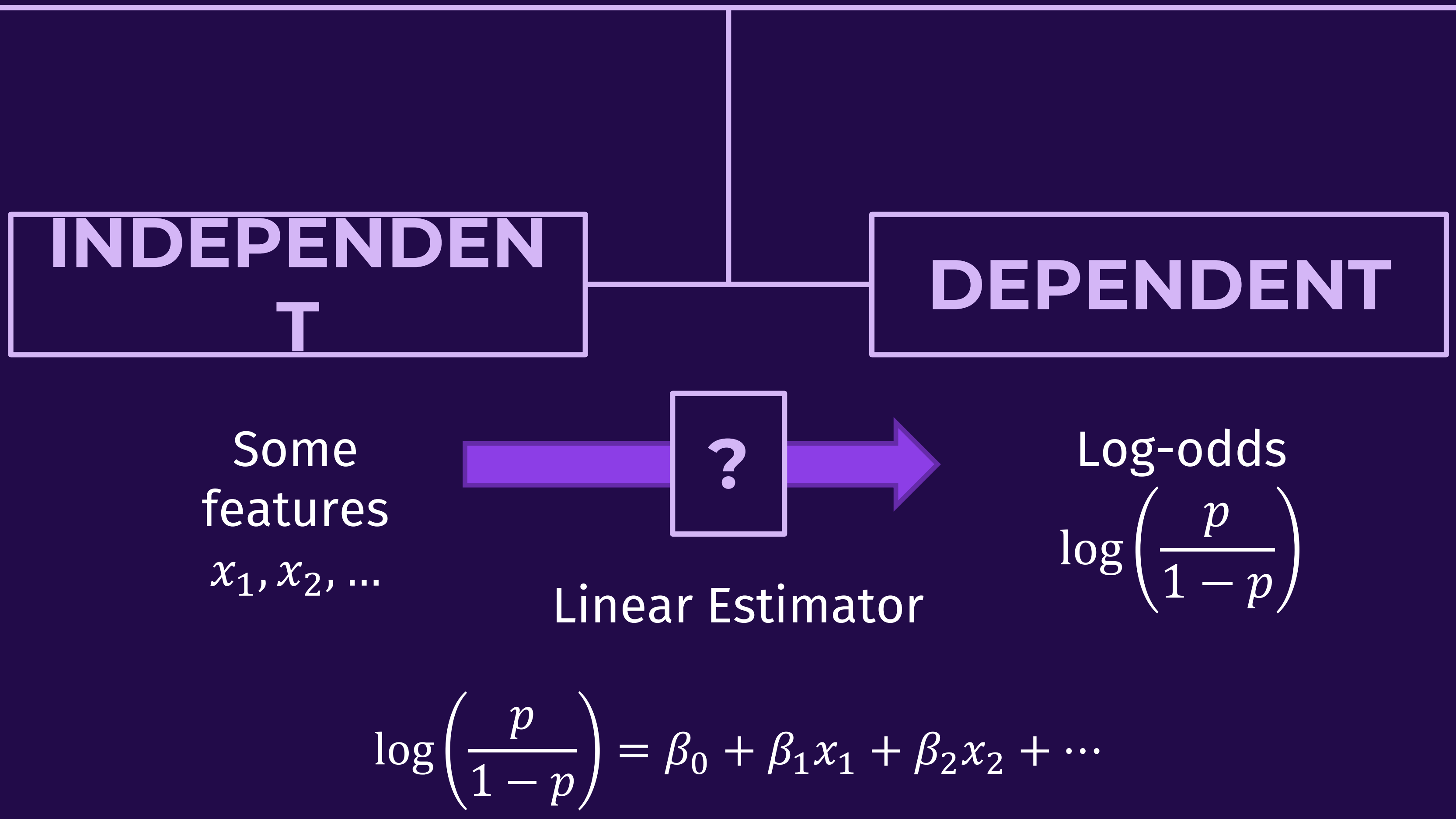


$$\log\left(\frac{p}{1-p}\right)$$

Log-odds

Odds:
likelihood of outcome

Log-odds:
logarithm of odds


INDEPENDENT
DEPENDENT
?
Some features $x_1, x_2, \ldots$
Linear Estimator
Log-odds $\log\left(\frac{p}{1-p}\right)$
$\log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots$

LOGISTIC = INVERSE OF THE LOG ODDS

$$\log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots$$

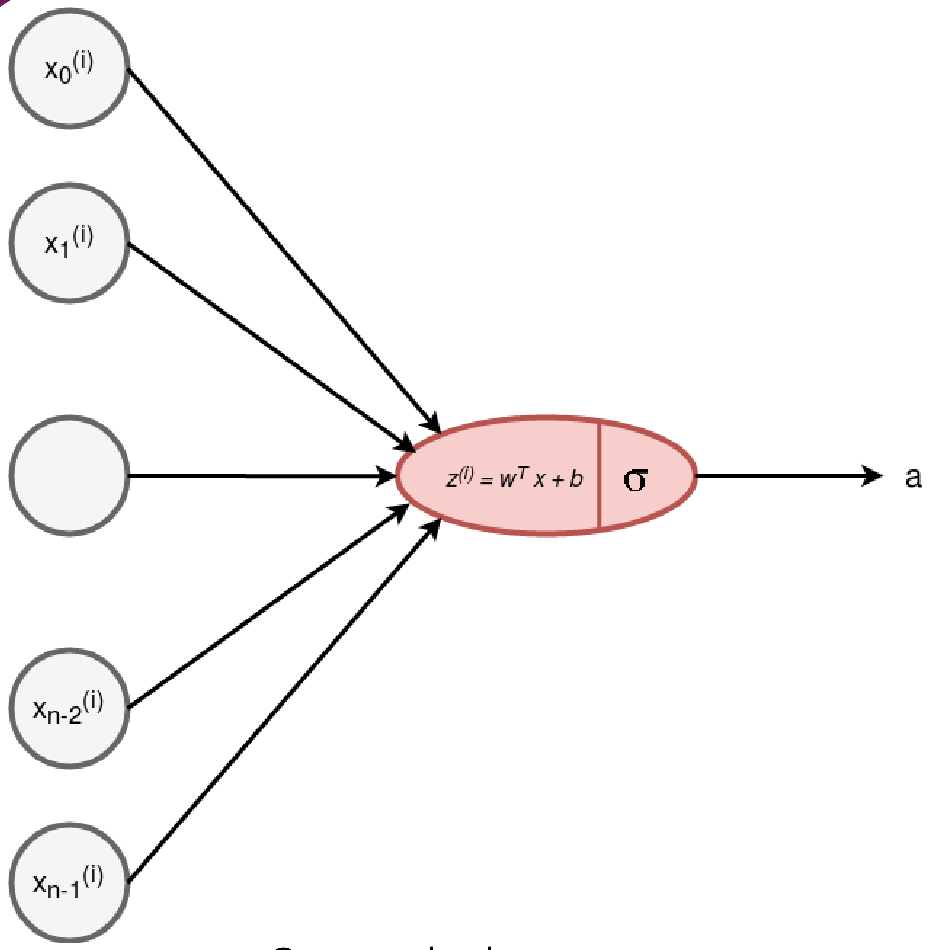
Inversed to

$$p = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots)}}$$

which is a logistic function!

WRAP UP

- We actually do "linear regression" then "activates" it using a logistic function
- Sometimes the activator is called **sigmoid function**
- Sigmoid is all S-shaped functions and logistic is just one of it.



Source: vbvsharma.com

LET'S GENERALIZE!

- Assume there are n outcomes.
- Let us choose a "pivot" outcome. For example, $Y = n$
- Instead using odds, we will just take the ratio to one "pivot" outcome.
- So, for each $Y = k \in \{1,2, \dots n-1\}$, we can regress its log-ratio:

$$\log\left(\frac{P(Y=k)}{P(Y=n))}\right) = \mathrm{B}_k X$$

- Note that $\mathrm{B}_k X = \beta_{k0} + \beta_{k1} x_1 + \cdots$

LET'S GENERALIZE!

- Thus, we have

$$\log\left(\frac{P(Y=1)}{P(Y=n))}\right) = \mathrm{B}_1 X$$

$$\log\left(\frac{P(Y=2)}{P(Y=n))}\right) = \mathrm{B}_2 X$$

$$\vdots$$

$$\log\left(\frac{P(Y=n-1)}{P(Y=1))}\right) = \mathrm{B}_{n-1} X$$

LET'S GENERALIZE!

- Little exponent manipulation yields

$$P(Y=1) = P(Y=n)e^{\mathrm{B}_2 X}$$
$$P(Y=2) = P(Y=n)e^{\mathrm{B}_3 X}$$
$$\vdots$$
$$P(Y=n-1) = P(Y=n)e^{\mathrm{B}_n X}$$

- +

$$\sum_{k=1}^{n-1} P(Y=k) = P(Y=n)\sum_{k=1}^{n-1} e^{\mathrm{B}_k X}$$

LET'S GENERALIZE!

- We know that

$$P(Y = n) = 1 - \sum_{k=1}^{n-1} P(Y = k) = 1 - P(Y = 1) \sum_{k=1}^{n-1} e^{\mathrm{B}_k X}$$

$$P(Y = n) = \frac{1}{1 + \sum_{k=1}^{n-1} e^{\mathrm{B}_k X}}$$

Thus,

$$P(Y = k) = \frac{\sum_{k=1}^{n-1} e^{\mathrm{B}_k X}}{1 + \sum_{k=1}^{n-1} e^{\mathrm{B}_k X}}, k \in \{1,2, \dots , n - 1\}$$

LET'S GENERALIZE!

- Actually $P(Y = n)$ can be known once the other $P(Y = k)$ have been obtained.
- This makes B_n can't be uniquely identifiable
- So we can transform a new parameter $\mathrm{B'}_k = \mathrm{B}_k - \mathrm{B}_n$ and that will yield to

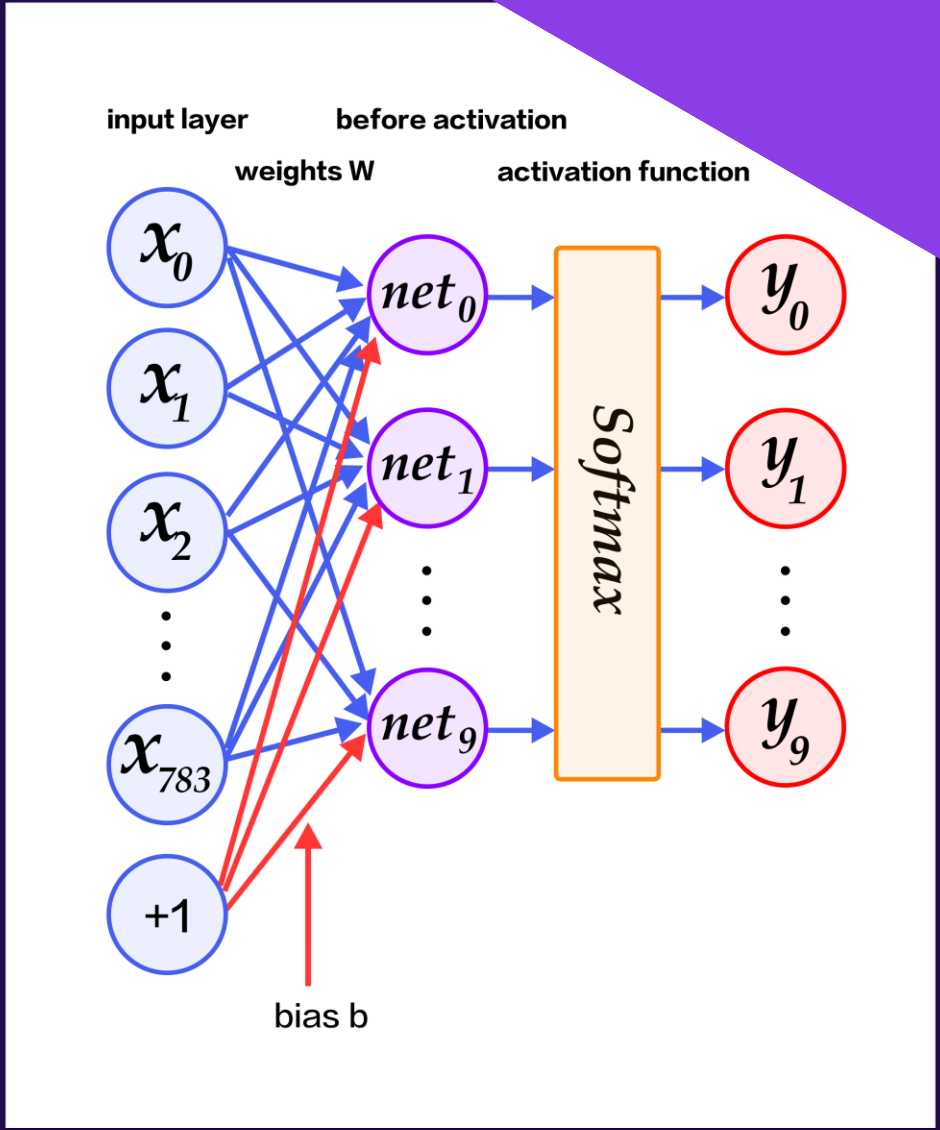
$$P(Y = k) = \frac{e^{\mathrm{B'}_k X}}{\sum_{k=1}^{n} e^{\mathrm{B'}_k X}}$$

$$P(Y = k) = \frac{e^{\mathrm{B'}_k X}}{\sum_{k=1}^{n} e^{\mathrm{B'}_k X}}$$

Yes, this is softmax function for categorical classification

WRAP UP

- If we have $k > 2$ category, we use softmax to activate the output.
- Softmax will be reduced to logistic function if $k = 2$



THANKS!

Do you have any questions?

aditya.fphoenix@gmail.com