



# AI di Pembelajaran Matematika

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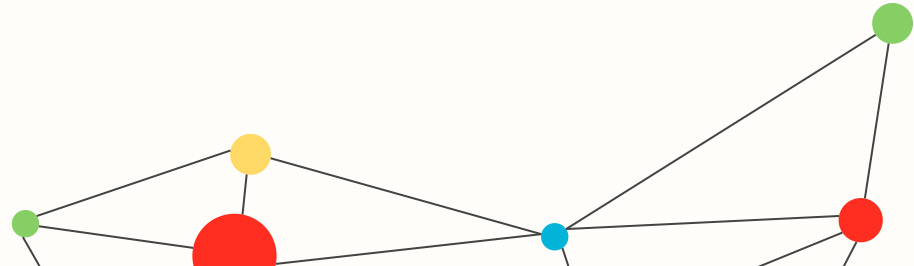

# Pendahuluan

Mathematics adalah ilmu yang rigid dan eksak.

Banyak siswa kesulitan memahami konsep abstrak dan mengaplikasikannya ke masalah-masalah kompleks

Masalah utamanya lebih sering pada kemampuan *problem-solving*, bukan pada kemampuan procedural

Metode pengajaran biasa sering terfokus pada pengetahuan procedural, yang cenderung akan membatasi kemampuan anak untuk mengembangkan kemampuan berpikir orde tinggi.





# Masalah umum di pembelajaran matematika

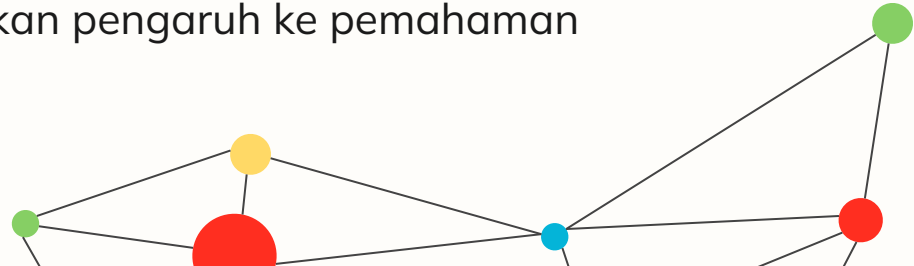
**Konsep abstrak** (Matematika bisa sangat abstrak dan sukar dibayangkan)

**Pacing dan keunikan siswa** (Kecepatan belajar anak bisa sangat berbeda)

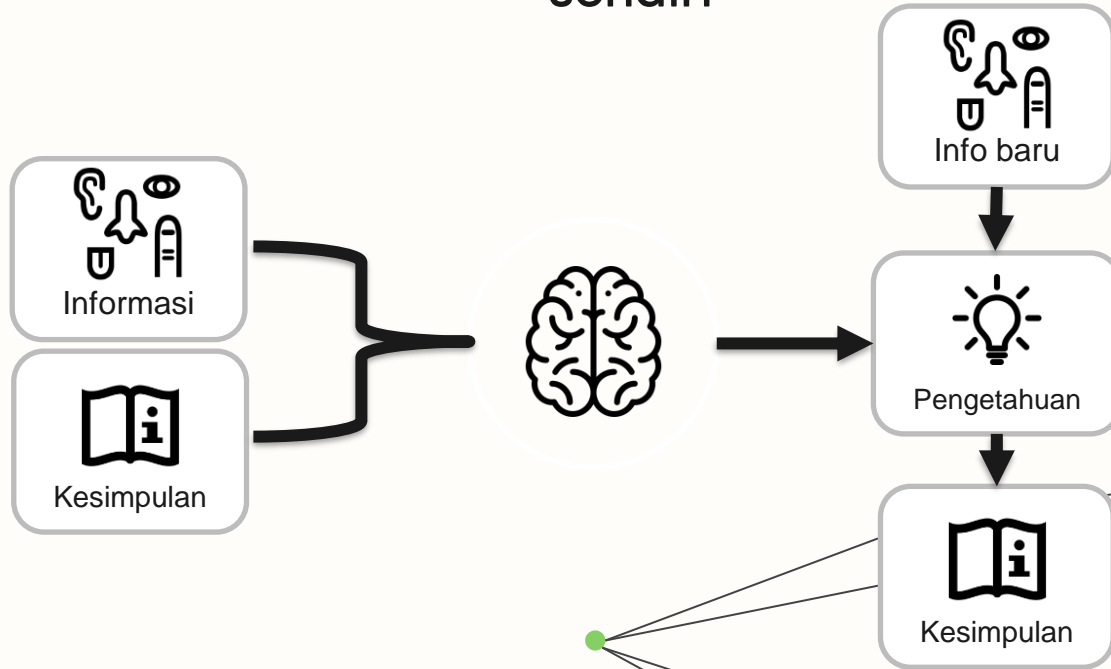
**Relevansi dunia nyata** (Siswa sering kesulitan melihat keterkaitan matematika dengan kesehariannya dan karir dia berikutnya)

**Metode asesmen** (ujian yang terstandarisasi tidak akurat memperlihatkan kemampuan siswa)

**Lemahnya fondasi** (matematika adalah ilmu yang sangat terstruktur, lemahnya pemahaman di satu hal akan pengaruh ke pemahaman berikutnya)

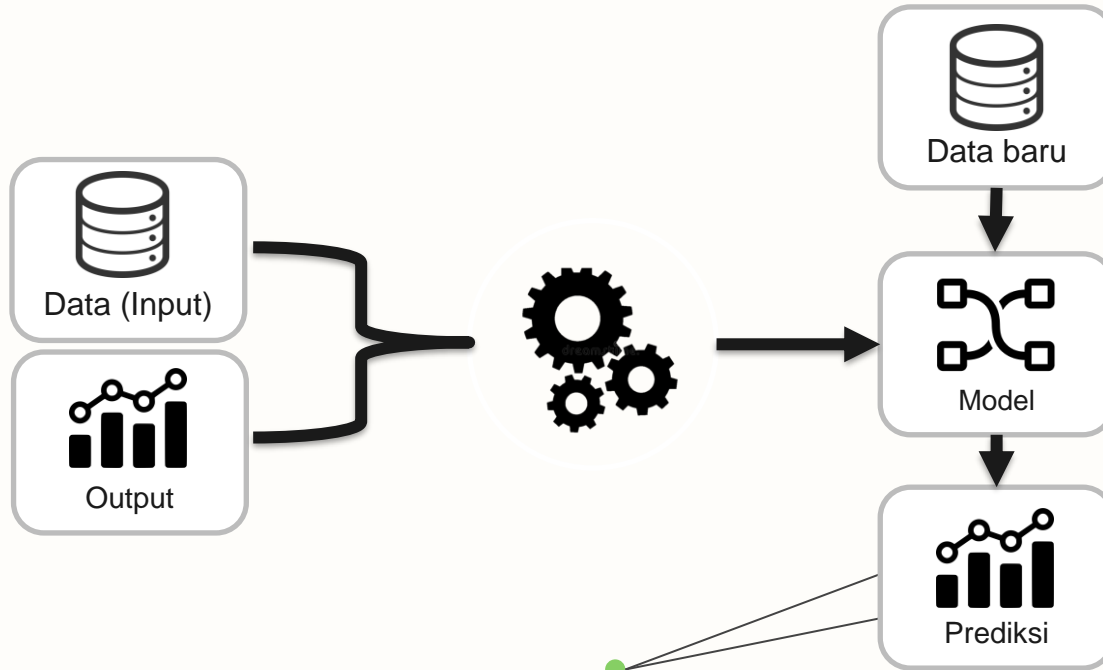


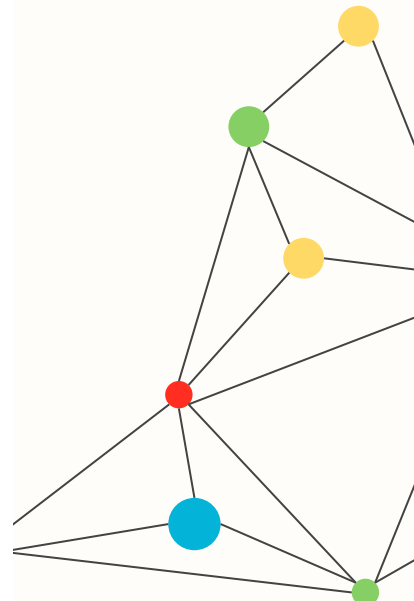
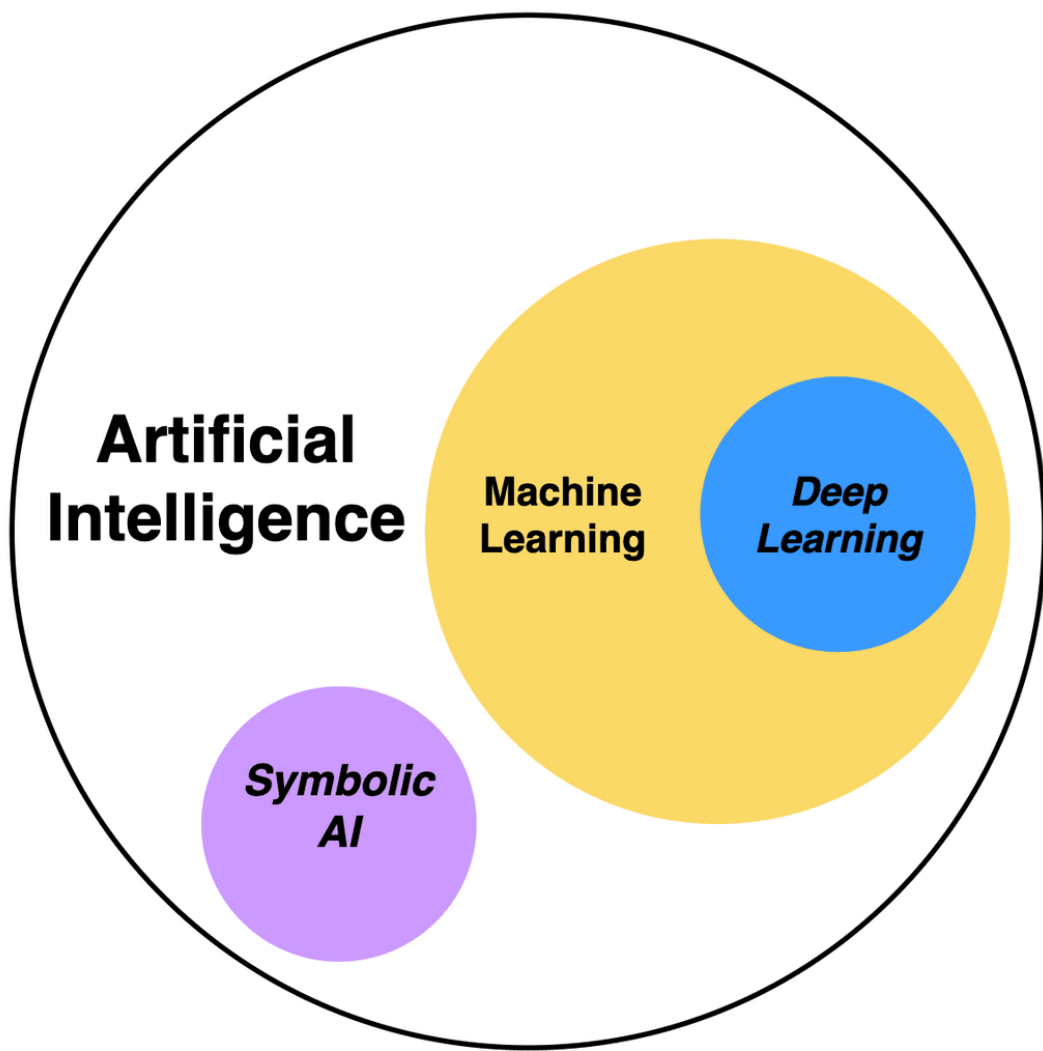
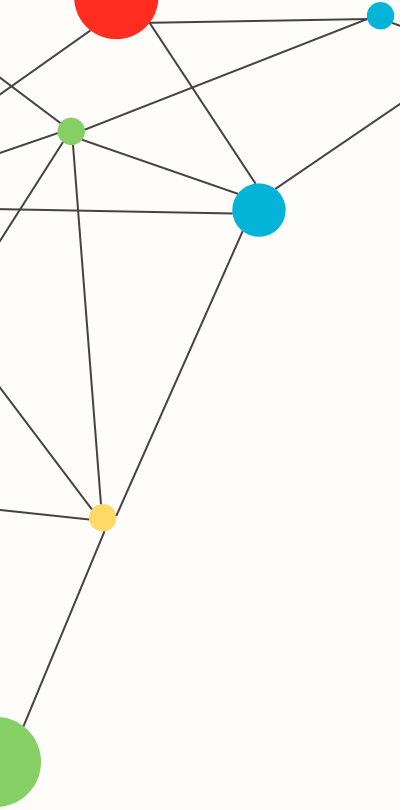
Manusia bisa mengembangkan pengetahuannya sehingga ketika diberi informasi baru, dapat disimpulkan sendiri



Demikian juga bila kita ingin buat mesin "belajar", mesin harus bisa mengembangkan "pengetahuan internalnya"

Ini disebut Machine Learning (ML)





Contoh Task

General

Teks

Visual

Klasifikasi

Clustering

Summarization

Penerjemah

Segmentasi

Deteksi Obyek

Deteksi Anomali

Sistem Rekomendasi

Generative

Question-Answering

Character Recognition

Style Transfer

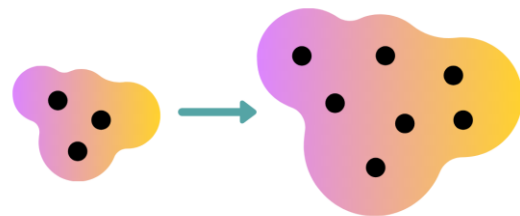
Sentiment Analysis



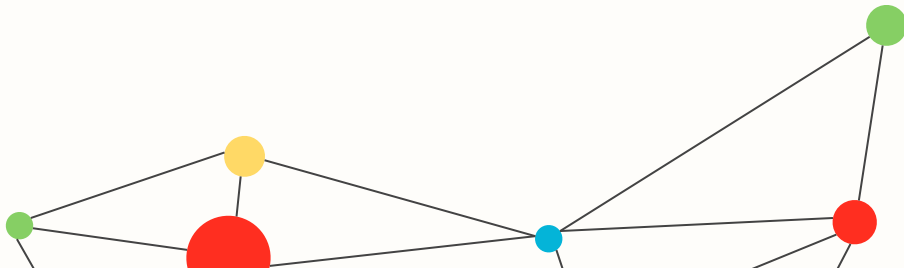
# Generative Model

Salah satu model ML adalah model generatif (model yang dapat “menciptakan”)

Model generative membaca pola data untuk membuat data baru

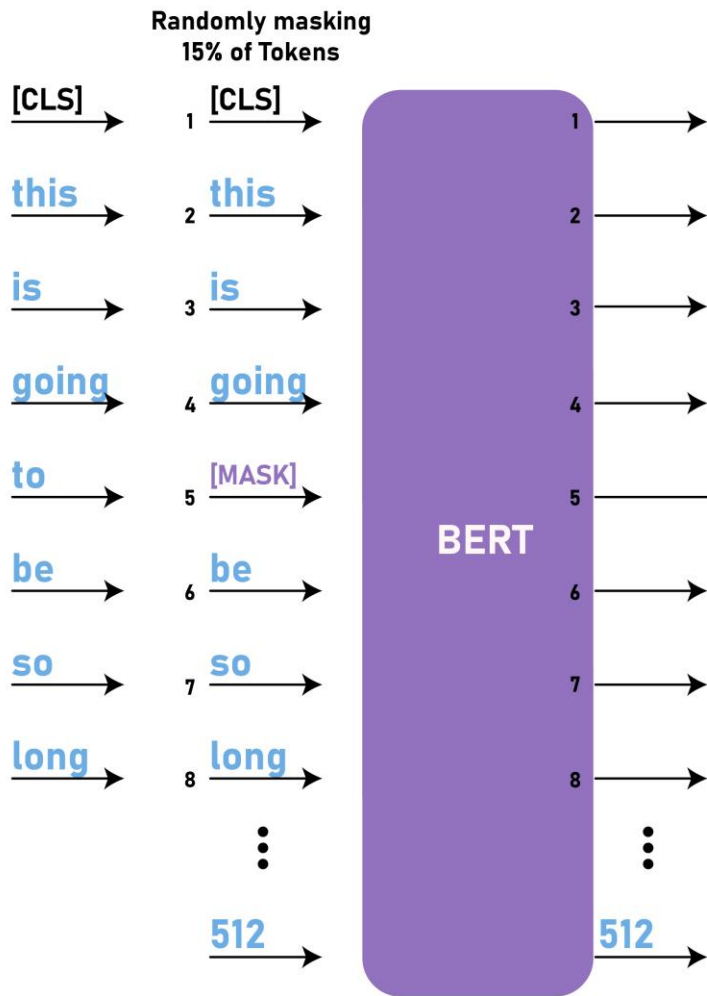


Generating new data using past data





INPUT



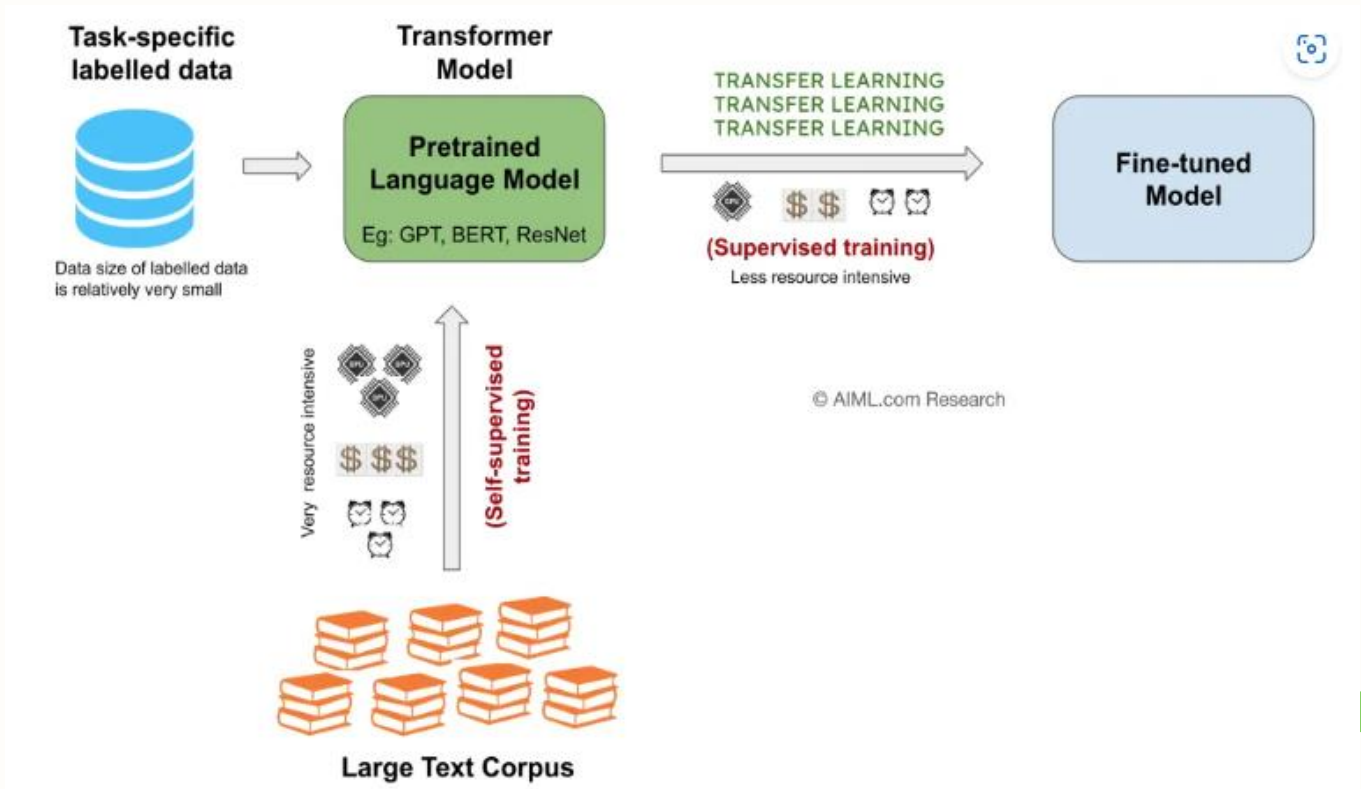
FFNN + SOFTMAX



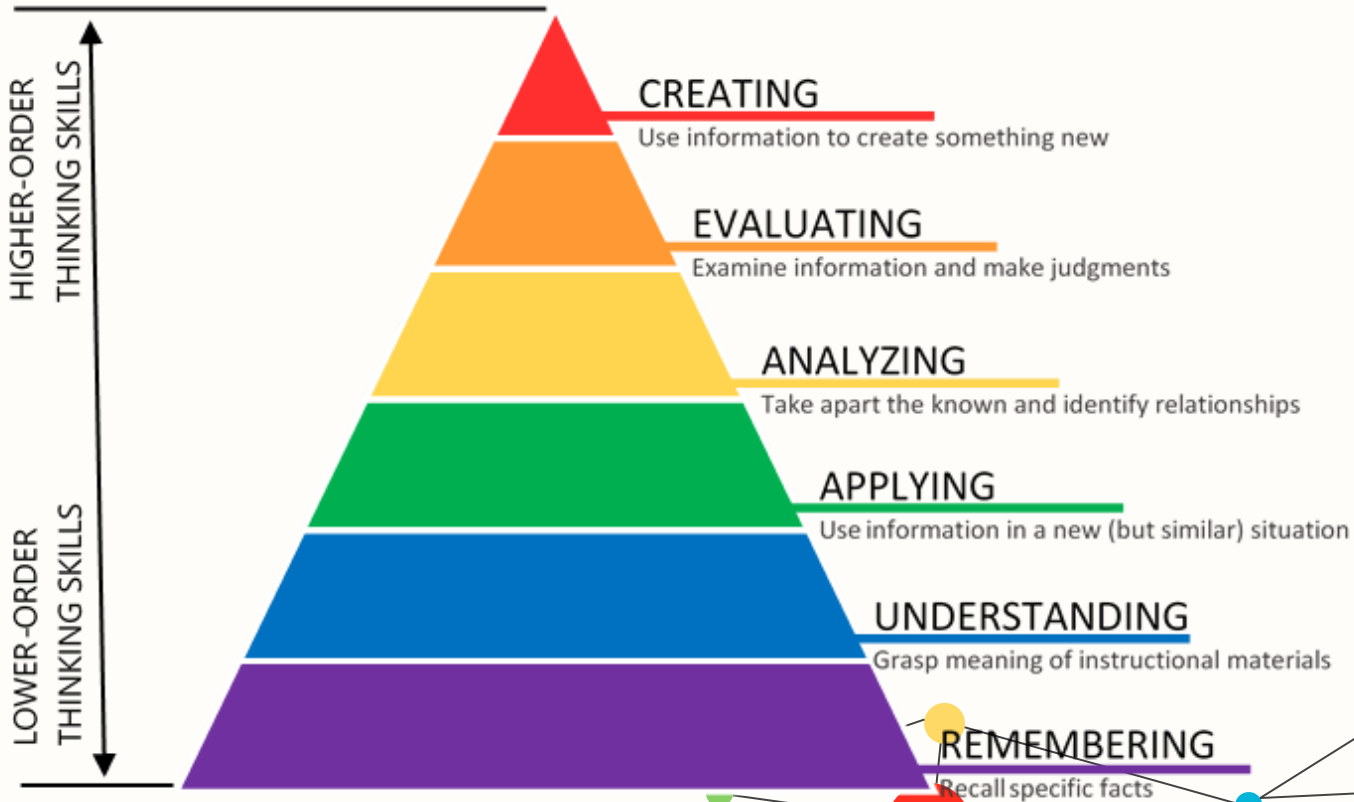
Using the output  
of masked word  
to predict that word

Khusus terkait teks, modelnya biasa disebut Language Model.

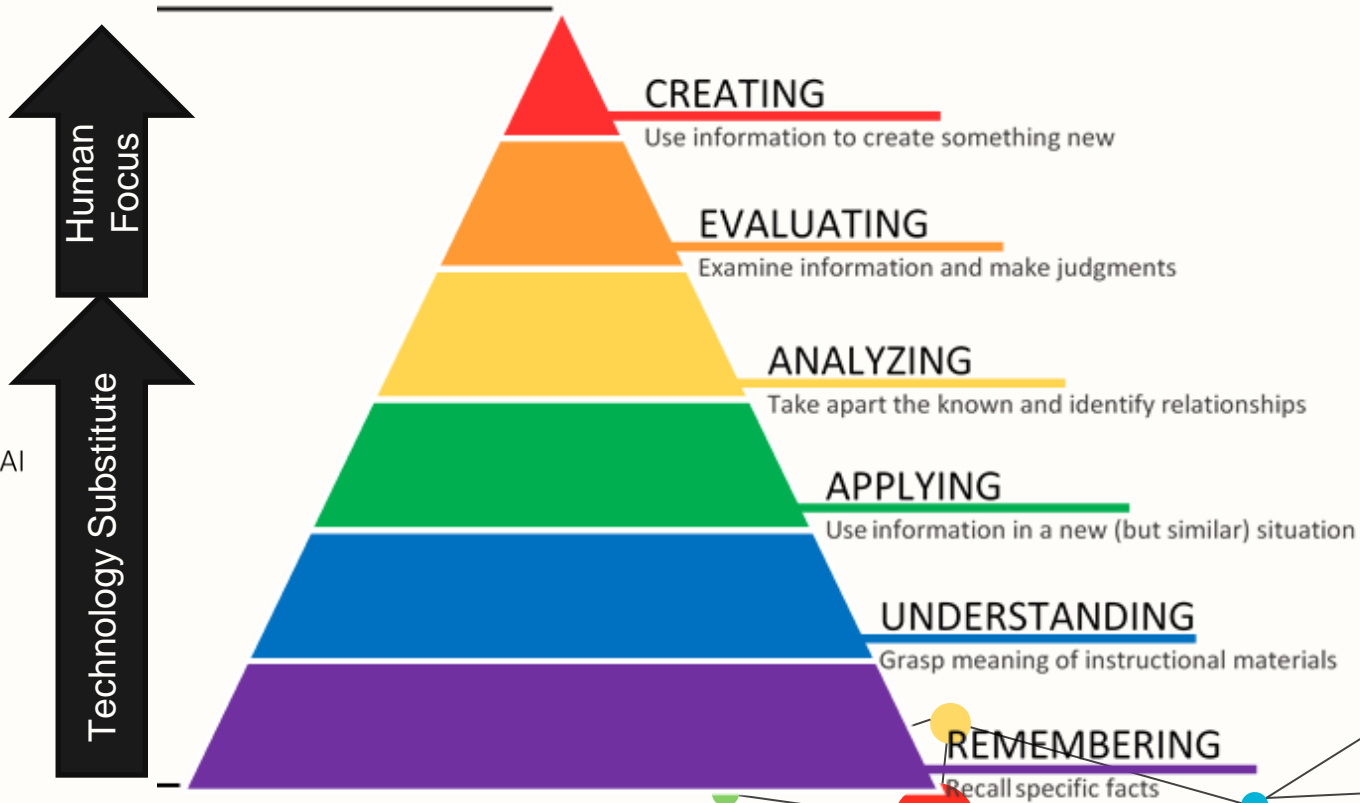
Sekarang, dengan data besar, terkenal dengan LLM (Large Language Model)



# Bloom's Taxonomy – Cognitive Domain (2001)



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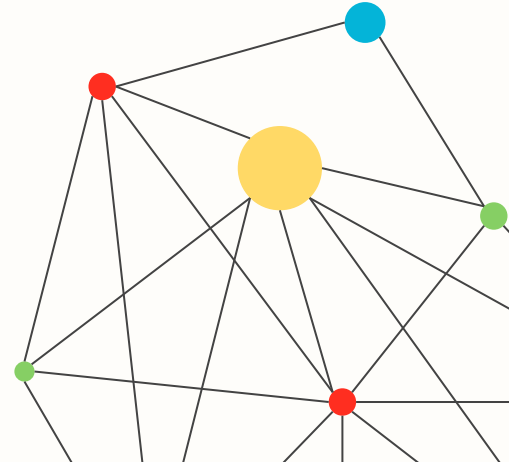

- GENERATIVE AI
- CALCULATOR
- WEBSITE
- WEB SEARCH



# ChatGPT

GPT (Generative Pre-Trained Transformer) adalah salah satu *language model*, dikembangkan oleh OpenAI.

Aplikasi yang dikembangkan di atas GPT ada banyak, salah satunya ChatGPT






# GPT untuk Matematika

ChatGPT dapat menyelesaikan masalah sederhana dan eksplisit dengan Solusi per langkah

## Prompt example:

*please solve derivative of  $5x \cdot \cos(x^2)$*



To solve the derivative of  $5x \cos(x^2)$ , we can use the product rule and the chain rule. Here's a step-by-step solution:

### Step-by-Step Solution

1. Identify the functions to apply the product rule:

The product rule states that for two functions  $u(x)$  and  $v(x)$ , the derivative of their product is:

$$\frac{d}{dx} [u(x) \cdot v(x)] = u'(x) \cdot v(x) + u(x) \cdot v'(x)$$

Here, let:

$$u(x) = 5x \quad \text{and} \quad v(x) = \cos(x^2)$$

...

...

...

4. Simplify the expression:

$$\frac{d}{dx} [5x \cos(x^2)] = 5 \cos(x^2) - 10x^2 \sin(x^2)$$

### Final Answer

The derivative of  $5x \cos(x^2)$  is:

$$5 \cos(x^2) - 10x^2 \sin(x^2)$$



# GPT untuk Matematika

2. ChatGPT dapat juga menyelesaikan soal cerita

## Prompt example:

*Please solve the following problem. A ladder 15 ft long rests against a vertical wall. Its top slides down the wall while its bottom moves away along the level ground at a speed of 2 ft/s. How fast is the angle between the top of the ladder and the wall changing when the angle is  $\pi/3$  radians*

Sure, I can help solve this problem using related rates, which involves calculus and the concept of limits.

### Problem Setup

We have a ladder of length  $L = 15$  ft resting against a vertical wall. The top of the ladder is sliding down the wall while the bottom is moving away from the wall along the ground. We are given that the speed at which the bottom of the ladder is moving away from the wall is  $\frac{dx}{dt} = 2$  ft/s. We need to find the rate at which the angle  $\theta$  between the ladder and the wall is changing when  $\theta = \frac{\pi}{3}$  radians.

### Variables and Relations

Let's define the variables:

- $x$  is the distance from the bottom of the ladder to the wall.
- $y$  is the height of the top of the ladder above the ground.
- $\theta$  is the angle between the ladder and the wall.

The relationship between  $x$ ,  $y$ , and  $L$  (the length of the ladder) can be described using the Pythagorean theorem:

$$x^2 + y^2 = L^2$$

Since  $L$  is constant (15 ft), we have:

$$x^2 + y^2 = 15^2 = 225$$

...  
...  
...

### Answer

The rate at which the angle between the top of the ladder and the wall is changing when the angle is  $\frac{\pi}{3}$  radians is approximately 0.27 rad/s.

# GPT untuk Matematika

3.a ChatGPT bisa memvisualisasikan fungsi dalam gambar statis.

## Prompt example:

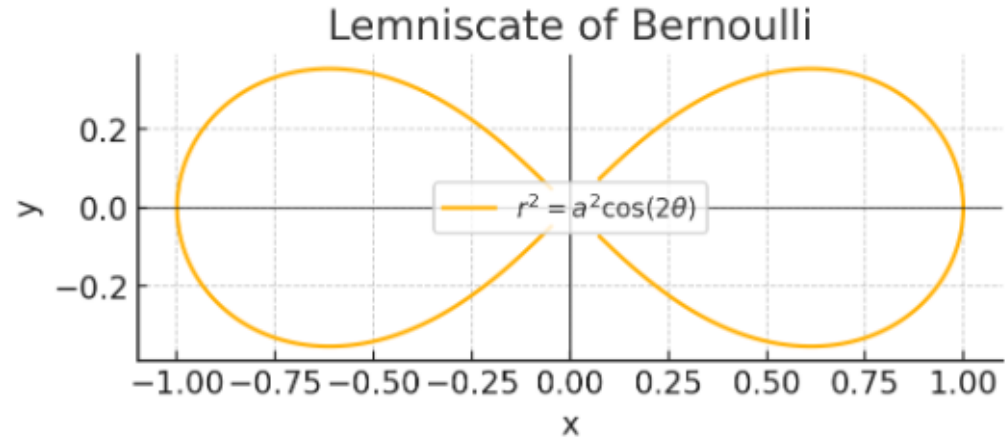
*I want you to draw a lemniscate*

A lemniscate is a figure-eight or infinity-shaped curve. One of the most famous lemniscates is the Lemniscate of Bernoulli, which can be described by the following equation in polar coordinates:

$$r^2 = a^2 \cos(2\theta)$$

where  $a$  is a constant that determines the size of the lemniscate.

Let's plot this curve. I'll choose a specific value for  $a$  to create the plot.



Here's the plot of the Lemniscate of Bernoulli. The figure-eight shape is characteristic of this type of curve. The equation used is  $r^2 = a^2 \cos(2\theta)$  with  $a = 1$ . The warnings about invalid values are expected since the cosine function can produce negative values, leading to complex numbers under the square root, which are ignored in this real-valued plot. [↔]



# GPT untuk Matematika

3.b ChatGPT bahkan juga menyediakan code apabila kita ingin mereplika gambarnya

## Prompt example:

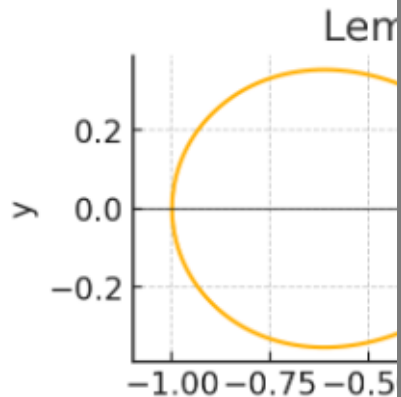
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where  $a$  is a constant that determines the size of the curve.

Let's plot this curve. I'll choose a specific value for  $a$ .



Analysis

```
python Always show details Copy code

import numpy as np
import matplotlib.pyplot as plt

# Define the value for the parameter 'a'
a = 1

# Create theta values
theta = np.linspace(0, 2 * np.pi, 1000)

# Calculate r values using the lemniscate equation
r = a * np.sqrt(np.cos(2 * theta))

# Convert polar coordinates to Cartesian coordinates for plotting
x = r * np.cos(theta)
y = r * np.sin(theta)

# Plot the lemniscate
plt.figure(figsize=(6, 6))
plt.plot(x, y, label=r'$r^2 = a^2 \cos(2\theta)$')
plt.title('Lemniscate of Bernoulli')
plt.xlabel('x')
```

Here's the plot of the Lemniscate of Bernoulli. The figure-eight shape is characteristic of this type of curve. The equation used is  $r^2 = a^2 \cos(2\theta)$  with  $a = 1$ . The warnings about invalid values are expected since the cosine function can produce negative values, leading to complex numbers under the square root, which are ignored in this real-valued plot. [?]



# ChatGPT in Math

4. ChatGPT dapat memberikan aplikasi yang terkait dengan suatu topik.

## Prompt example:

*I am a computer science student trying to understand concept of derivatives. Can you give me a related real-world example that have relevance to my major?*

Certainly! Understanding derivatives in the context of computer science can be incredibly useful, especially in fields like machine learning, optimization, and computer graphics. Let's explore an example involving machine learning, which is highly relevant to your major.

## Example: Gradient Descent in Machine Learning

### Problem Context:

Suppose you are training a machine learning model, such as a linear regression model, to predict housing prices based on various features (e.g., size, number of bedrooms, location). The goal is to find the best-fit line that minimizes the error between the predicted prices and the actual prices.

### Derivatives in Action:

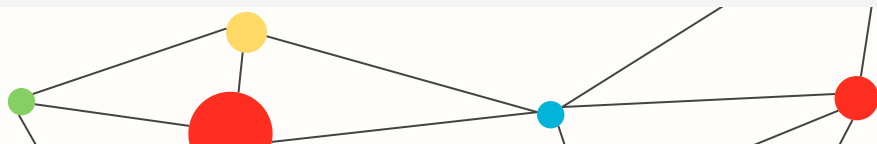
#### 1. Cost Function:

To quantify the error, you use a cost function, such as the Mean Squared Error (MSE):

$$\text{MSE}(\theta) = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

where:

- $m$  is the number of training examples.





# Possible Utilization of GPT in Math

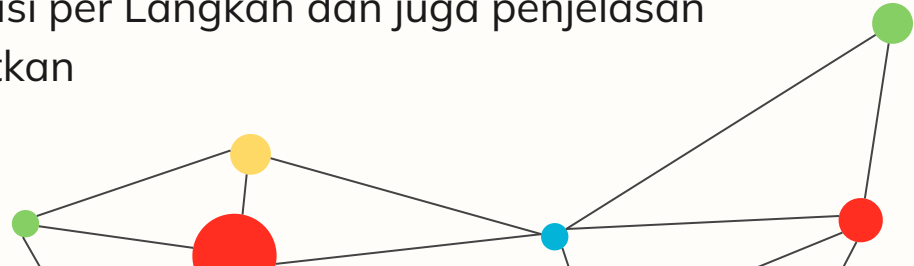
**Konsep abstrak** -> memberikan penjelasan, visualisasi, dan lebih banyak contoh dari konsep yang abstrak

**Pacing dan keunikan siswa** -> memberikan soal-soal latihan yang lebih disesuaikan dengan kemampuan siswa

**Relevansi dunia nyata** -> memberikan bagaimana matematika digunakan di keseharian dan berbagai karir

**Metode asesmen** -> feedback langsung dari jawaban siswa, sehingga dapat langsung dikoreksi dengan penjelasan

**Lemahnya fondasi** -> menyediakan Solusi per Langkah dan juga penjelasan atas konsep yang masih perlu dikuatkan



# Prompting

- Because the nature of math is concise and exact, ChatGPT is so useful in terms of direct math.
- It does not need long prompt. Just give it direct question.
- It does not have tendency to give wrong answers

# Prompts for students

Dalam hal ini, guru sebagai fasilitator bisa dengan sengaja mengarahkan siswa untuk menggunakan GPT.

Yang dapat ditanyakan adalah:

1. Pemahaman dasar,
2. Contoh soal
3. Aplikasi spesifik

Setiap siswa kemudian diminta untuk lebih lanjut eksplorasi dari promptnya

# Prompts for students

## Limits and Continuity

- **Basic Understanding:** "Explain the concept of a limit in calculus."
- **Example Problem:** "Find the limit of  $(2x^2 - 3x + 1)$  as  $x$  approaches 3."
- **Continuity:** "Explain what it means for a function to be continuous at a point."

## Derivatives

- **Basic Understanding:** "What is a derivative, and how is it different from a function?"
- **Example Problem:** "Differentiate the function  $f(x) = 3x^3 - 5x^2 + 6x - 2$ ."
- **Application:** "Explain how derivatives are used to find the slope of a tangent line to a curve."

## Integrals

- **Basic Understanding:** "What is an integral, and how is it different from a derivative?"
- **Example Problem:** "Compute the integral of  $4x^3 - 2x + 1$  with respect to  $x$ ."
- **Application:** "How are integrals used to find the area under a curve?"

## Series and Sequences

- **Basic Understanding:** "What is the difference between a sequence and a series in calculus?"
- **Example Problem:** "Determine if the series  $\sum (1/n^2)$  from  $n=1$  to infinity converges."
- **Convergence Tests:** "Explain the ratio test for convergence of a series."

# Lalu apa peran pengajar?

Pengajar dapat cukup menjadi fasilitator ketimbang yang secara satu arah memberi materi.

1. Mengawasi perkembangan dan pemahaman siswa
2. Memastikan setiap penggunaan AI yang dilakukan tepat
3. Mencegah siswa untuk langsung ke jawaban akhir ketika berusaha menyelesaikan soal
4. Dorong siswa untuk Menyusun prompt yang tepat ketika mereka tidak paham beberapa poin
5. Validasi respon dari ChatGPT
6. Berikan materi tambahan dari apa yang sudah diberikan ChatGPT

# Contoh mekanisme mengajar (di kampus)

- Di awal semester, dosen memberikan gambaran umum tentang seluruh materi.
- Untuk setiap topik, dosen menjelaskan apa yang akan dipelajari, konteks, latar belakang, dan motivasi dari topik tersebut.
- Dosen memberi instruksi kepada mahasiswa untuk berinteraksi dengan ChatGPT guna mengeksplorasi materi lebih lanjut.
- Dosen memberikan beberapa contoh untuk dikerjakan oleh mahasiswa di depan kelas. (sesi kelas ini akan sangat bergantung pada kegiatan ini)
- Penilaian dapat dilakukan dengan mengevaluasi proses diskusi sepanjang kuliah (karena matematika sebaiknya dinilai berdasarkan kemampuan pemecahan masalah individu).



# Evaluasi

Paling tidak ada 2 aspek kemampuan matematika

## Komputasi

Menyelesaikan secara langsung soal eksplisit

## Problem Solving

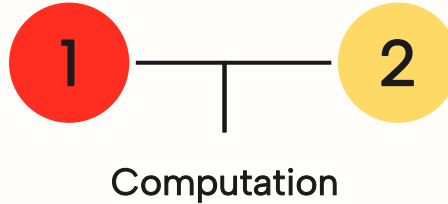
Informasi parsial yang membutuhkan konstruksi ulang soalnya dalam bentuk yang lebih eksplisit

# Math Measures

4 ukuran (measure) yang terkait

## Kecakapan fakta

Dapat mengulang fakta dari konsep yang dipahami



## Komputasi prosedural

Soal dengan prosedur yang jelas dan langsung untuk dihitung

Problem Solving

## Simple Word Problem

Soal yang membutuhkan langkah tambahan untuk dihitung



## Complex Word Problem

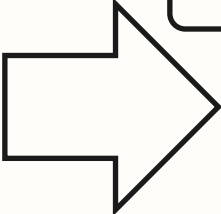
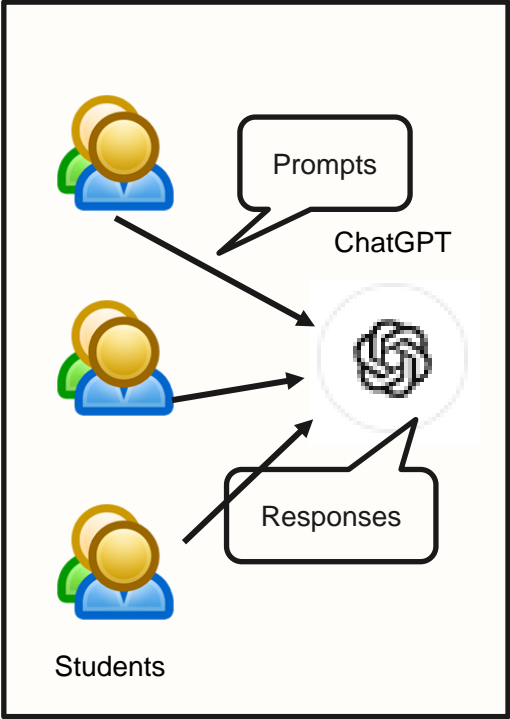
Soal dengan cerita dan konteks yang butuh dipahami dan diartikan lebih lanjut

# Math Measures

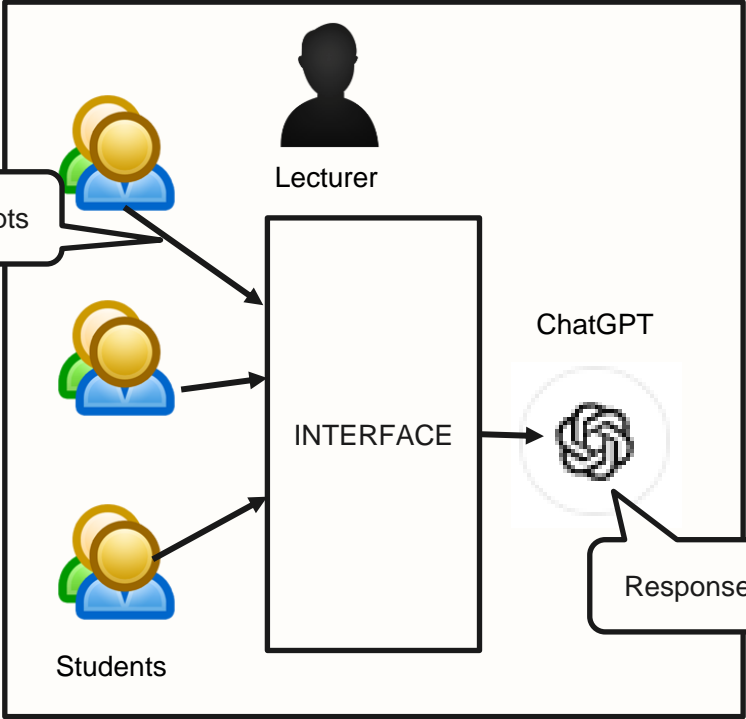
Aspect	Math Measures	Sample Questions
Computational	Fact Fluency	One requirement to apply quotient rule of derivatives is ....
	Procedural computation	Find $D_x$ of $y = \pi x^3$
Problem Solving	Algorithmic word problem	Find the equation of the tangent line to $y = x^2 - 2x + 2$ at the point (1,1)
	Complex word problem	A space traveler is moving from left to right along the curve $y = x^2$ . When she shut off the engines, she will continue traveling along the tangent line at the point where she is at that time. At what point should she shut off the engines in order to reach the point (4,15)?

# Opportunity for Improvement

BEFORE



AFTER



Selain ChatGPT, ada banyak alat berbasis AI lainnya yang sudah berkembang, namun rata-rata semuanya pada dasarnya berbasis GPT



Julius



Mathos AI  
Your Only Math Solver



Socratic




# Beberapa Catatan


Siswa tetap butuh sosok manusia untuk tempat bertanya

Pengajar dapat lebih fokus untuk membangun motivasi siswa

Material pembelajaran sudah terlalu banyak ada di Internet dan AI



Tanpa perlu diminta pun, anak-anak sebenarnya sudah mulai pakai AI, sehingga lebih baik diarahkan dari awal.





# Thanks!



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